A SPECTRAL STUDY OF THE WARMING EPOCH OF JANUARY–FEBRUARY 1958*

SIDNEY TEWELES

U.S. Weather Bureau, Washington, D.C.

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ABSTRACT

Contour and isotherm patterns of 100-, 50-, and 30-mb. charts have been subjected to harmonic analysis after the manner of Saltzman and Fleisher. The resulting wave-number statistics permit a detailed examination of the sudden warming and circulation breakdown that occurred in the Northern Hemisphere stratosphere in January-February 1958.

The stratospheric warming epoch of January 1958 was preceded by a marked expansion in the ring of tropospheric westerlies. Thereafter in the stratosphere the kinetic energy of wave number 1 increased by nearly the same amount transferred to it by the zonal flow and the other waves. However a subsequent great increase in the kinetic energy of wave number 2 occurred simultaneously with a large transfer of kinetic energy from that wave to both the zonal flow and the other waves. This development of wave number 2 thus appears to have been baroclinic in nature. Correlation of daily spectral statistics for the stratosphere and troposphere show a number of significant interlevel relationships in the growth and decay of the longer cyclone waves.

1. INTRODUCTION

In 1958 the late Dr. Harry Wexler assigned to the writer the task of organizing the Stratospheric Meteorology Research Project (SMRP) to prepare a series of stratospheric charts and perform research on the upper atmosphere. By the time the map analysis group was phased out in 1962, daily 100-mb. and 50-mb. charts and three-times-monthly 30-mb. charts for the 24 months, July 1957 to June 1959 (U.S. Weather Bureau [15]) had been completed. Grid values of contour heights read from these charts by the U.S. Navy Weather Research Facility and similar values read from 500-mb. charts by the U.S. Air Force 433-L Project permit making a detailed analysis of the circulation during the course of a sudden warming of the stratosphere in January-February 1958. The method by which the data are subjected to one-dimensional Fourier analysis for resolution of atmospheric energetics by wave number was developed by Saltzman [9]. This method after being programmed for the electronic computer, has been applied to the 500-mb. level by Saltzman and Fleisher [10, 11, 12]. Results obtained by adapting their program for use with data from SMRP stratospheric charts are contained in an M.I.T. Planetary Circulations Project Report (Teweles [14]). The portion of the M.I.T. Report adapted for publication here deals not only with the kinetic energy exchange by wave number but also with many of the simpler harmonic quantities that appear as intermediate steps in the energy computations. Emphasis here is on daily values, but many additional monthly mean values may be found in [14].

The decisive break in stratospheric circulation in the second half of January 1958, when viewed in its day-to-day progress on Northern Hemisphere charts at 50, 30, and 10 mb. (Freie Universität Berlin [4]; U.S. Weather Bureau [15]), is most impressive as a wave phenomenon. The major changes are obviously those of the zonal current and waves 1 and 2; thus, it is interesting to examine the energetics of this development in the domain of wave number. One such effort has already been made by Nishimoto [8] in his spectral analysis of the energetics at 25 mb. on January 21 and 27, 1958. However a complex series of events occurred during a period of several weeks, and a presentation of the daily course of events is desirable.

In an attempt to isolate the type of instability responsible for rapid development of stratospheric systems, Murray [7] developed mathematical models having some of the principal characteristics of the stratosphere. Although he was led to speculate that barotropic instability was the most fruitful possibility for further investigation, less artificial models than those he subjected to dynamic analysis may demonstrate conditions under which baroclinic instability can be released. From spectral resolution of the stratospheric events in the 1958–59 winter, Boville [1] concluded that baroclinic instability was an important factor in periods of rapid development. Fleagle's [3] criteria, derived from linearized equations, indicate baroclinic instability in very long waves at high latitudes for conditions of static stability and vertical wind shear like those at the time of the breakdown.

In the rapid development of wave 2 during the week following January 18, 1958, baroclinic instability appears

*This paper is extracted from dissertation [14] which was submitted in partial fulfillment of the requirements for Ph. D. degree, Massachussetts Institute of Technology, 1933.

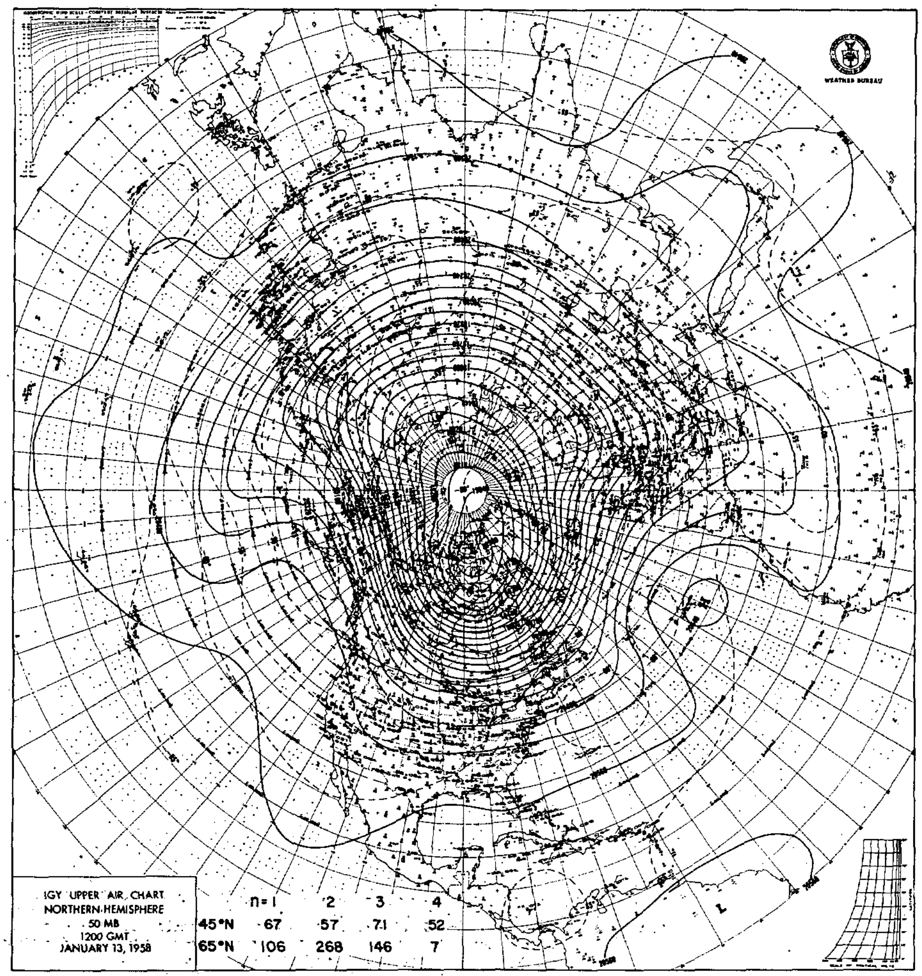

FIGURE 1.—Northern Hemisphere 50-mb. chart for 1200 GMT, January 13, 1958 (from [15]). Contours are given in meters, isotherms in °C. Amplitudes of wave numbers 1–4 at 45° N. and 65° N. are tabulated in meters at the bottom of the map.

to have been operative. However, finding instances when critical values for instability parameters are satisfied may not be as difficult as finding the stabilizing influences that come into play on the many occasions when stratospheric perturbations begin to develop only to subside again.

2. WAVE NUMBER IN DAILY CHARTS

As a background for the discussion to follow, the 50-mb. charts for January 13, 19, 25, and 31, 1958 (U.S. Weather Bureau [15]) are reproduced here (figs. 1–4). The amplitude (in meters) of waves 1–4 at 45° N. and at 65° N. is entered on each chart for reference.

The 50-mb. circulation on January 13 (fig. 1) is typical of that which existed from the beginning of the month with waves of moderate amplitude superimposed on a strong zonal current. By January 19 (fig. 2) wave 1 at 65° N. had amplified by a factor of more than six. Most of this amplification was merely due to movement of the circulation center away from the pole toward northern Europe. The picture is unusually clearcut because of the low amplitude of the other waves.

The map of January 25 (fig. 3) is an example of high amplitude in wave 2 with wave 1 still strong at 65° N. This interesting pattern is the result of reinforcement of ridges of the two waves near 135° W. and interference between the trough in wave 1 and second ridge of wave 2 on the opposite side of the pole, while the troughs of wave 2 formed a dumbell-shaped Low stretched across the eastern Arctic. Note that wave 1 exceeded wave 2 in strength at 65° N. but had little amplitude at 45° N.

By January 31 (fig. 4) wave 1 had regained strength particularly at 45° N. where wave 2 had weakened by about the same amount. The phase relation was such that the northward kinetic energy transport in the powerful current near 150° E. greatly exceeded the southward transport in the region between 50° N. and the pole elsewhere in the hemisphere; this feature accounts for the large transport by transient eddies at 50 mb. in January.

3. WAVE NUMBER RESOLUTION OF THE SYNOPTIC CHARTS

In a sense, Fourier analysis can be considered as removing the individual waves from the contour chart one by one. The height along a latitude circle given as a function

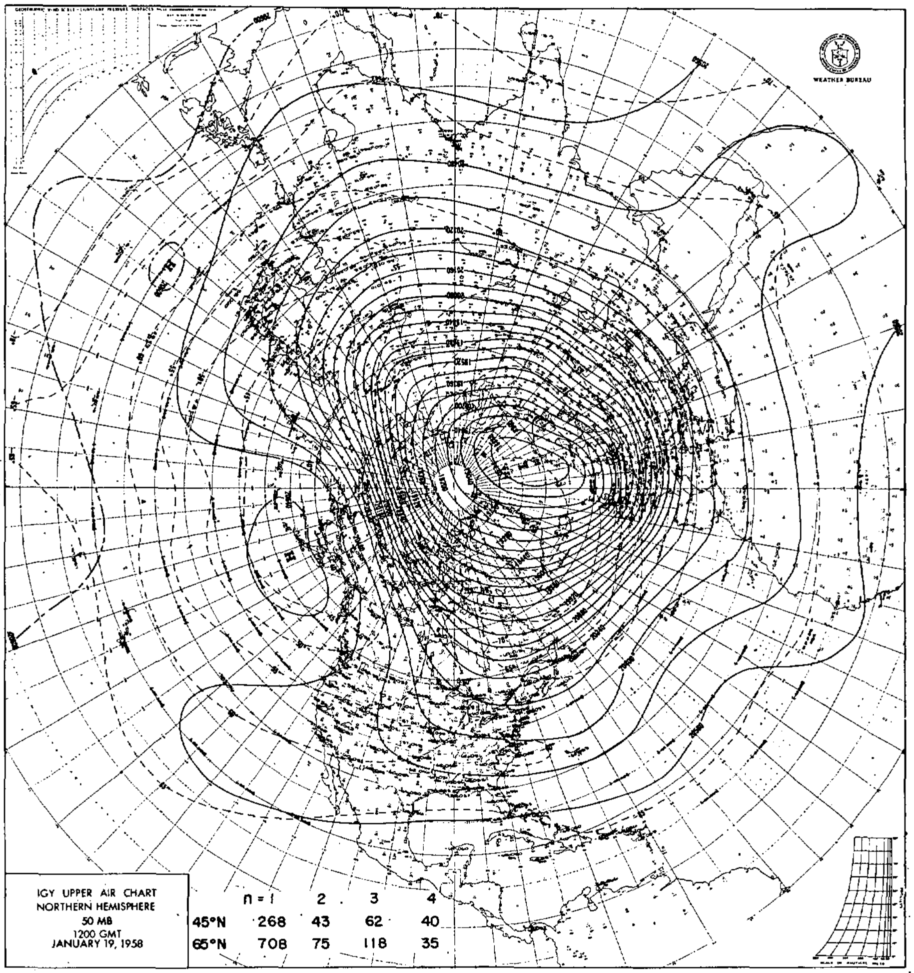

FIGURE 2.—50-mb. chart for January 19, 1958, as in figure 1.

of longitude $z(\lambda)$ may be resolved to give a zonal harmonic analysis with an amplitude and phase for each of the wave numbers of interest (see, for example, Saltzman [9]).

To serve as examples, the 50- and 500-mb. contour patterns for each day of January 1958 have been resolved into component waves. The phase and amplitude of the stationary waves may be obtained from a direct resolution of the monthly mean chart.

For wave 0 the amplitude is equivalent to the mean height around each latitude. Contours drawn for these zonally-averaged heights at 50 mb. (fig. 5) and at 500 mb. (fig. 6) represent the background circumpolar vortex upon which the transient eddies and stationary waves for the month are superimposed. The concentration of contour gradients at the latitude of the jet stream is clearly shown; however, the gradient across the jet at 50 mb. is deceptively tight since the Coriolis parameter there is almost double that at the latitude of the 500-mb. wind maximum.

Selected harmonics extracted from the mean monthly circumpolar flow for January 1958 are shown in figures 7, 8, and 9. Since the wave amplitude tends to diminish rapidly with wave number, the contour gradient of each chart has been scaled to bring out its features.

At 50 mb. (fig. 7) only waves 1–4 seem to have real meaning in terms of daily map patterns. Intense high cells in both waves 1 and 2 augmented one another in the Alaskan area, but over Europe the second High of wave 2 tended to be nullified by the Low in wave 1. The two Lows of wave 2 were situated over relatively cold regions of the hemisphere. Waves 3 and 4 were of small amplitude; however in middle latitudes the marked northeastward tilt of stationary wave 4 allowed it to transport poleward an amount of angular momentum disproportionate to its amplitude.

At 500 mb. (fig. 8) wave 1 was positioned 130° to the east and 15° to the south of its counterpart at 50 mb. Thus the monthly mean positions of wave 1 in the troposphere and stratosphere lacked only 50° longitude of being completely out of phase. According to the normal 500-mb. wave number charts of Van Mieghem, Defrise, and Van Isacker [16], the 500-mb. wave 1 for January 1958 was located slightly to the northeast of its normal position. Comparison of the 50-mb. mean chart for this

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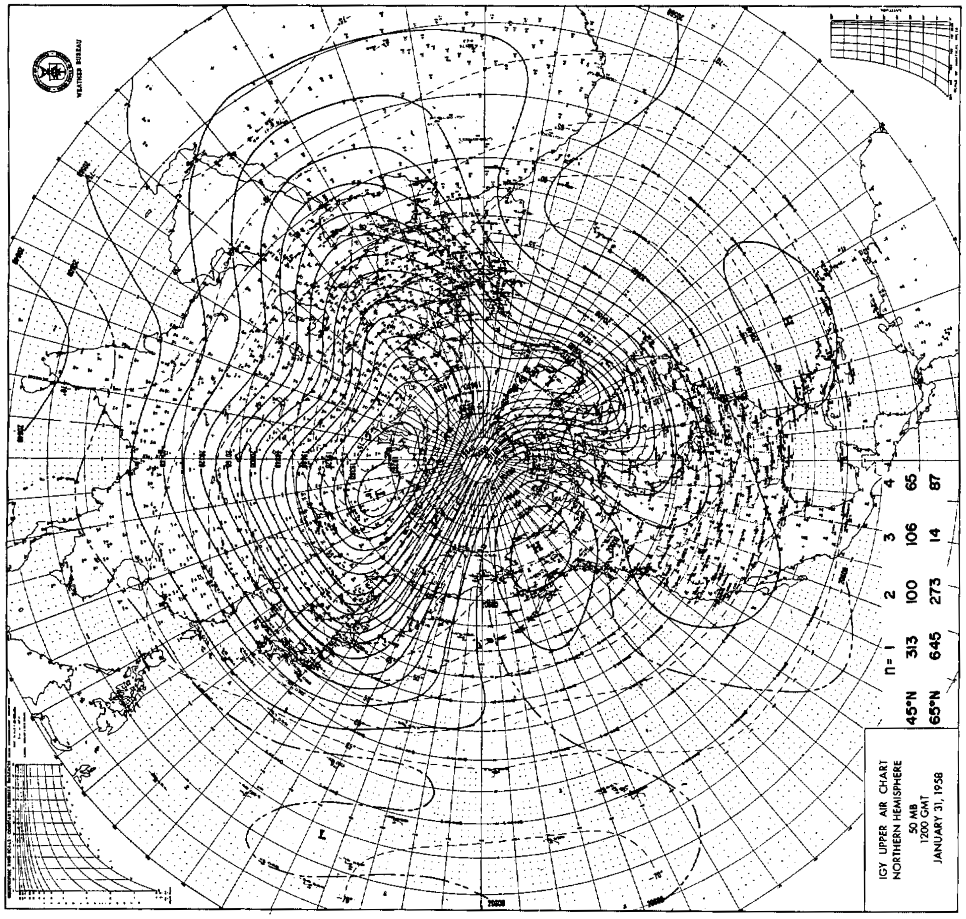


FIGURE 4.—50-mb. chart for January 31, 1958, as in figure 1.

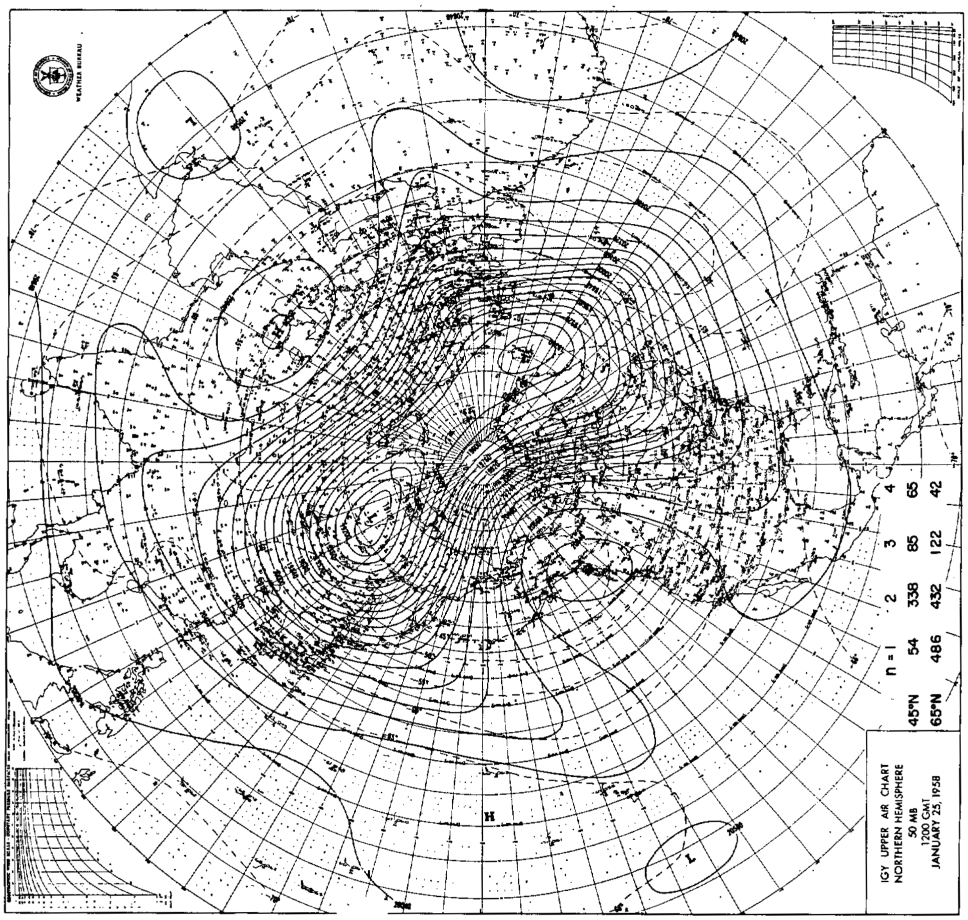


FIGURE 3.—50-mb. chart for January 25, 1958, as in figure 1.

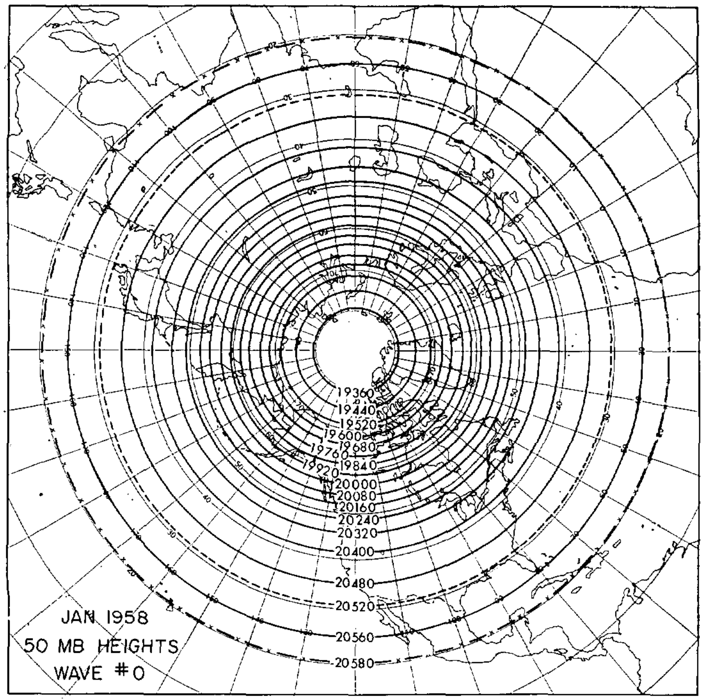


FIGURE 5.—January 1958 50-mb. standing wave number 0, the monthly and zonally-averaged height of the pressure surface, in meters.

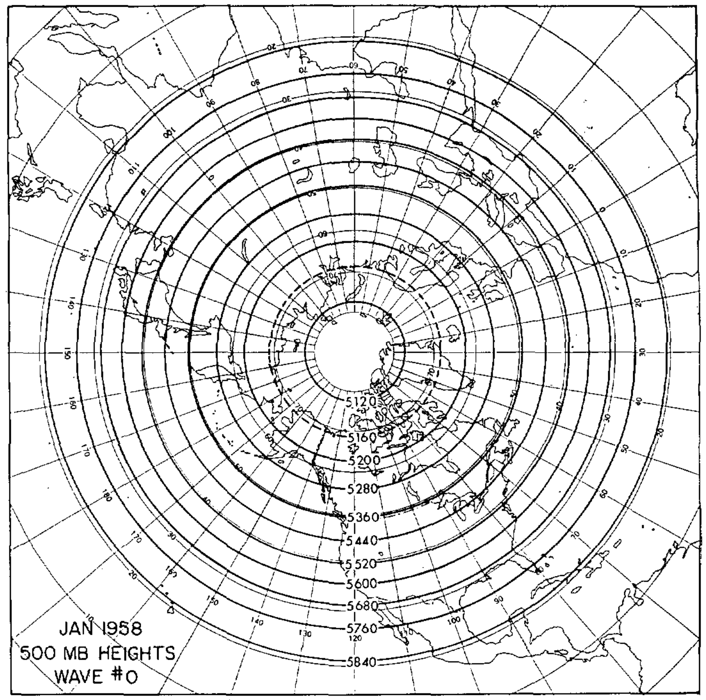


FIGURE 6.—January 1958 500-mb. standing wave 0 as in figure 5.

month with a 4-yr. January mean (Muench [6]) indicates that in 1958 wave 1 at 50 mb. was a short distance east of the mean position. Thus the peculiar vertical slope of the stationary wave 1 from 500 mb. up to 50 mb. observed in January 1958 seems to be a permanent feature, but figures 10 and 11 show large day-to-day deviations in this slope especially at 65° N.

Although a qualitative explanation for the differences between the eccentricities of the flow in the stratosphere and troposphere has been proposed (Teweles [13], pp. 394–396), the phenomenon remains to be explained dynamically by means of a realistic model. That there can be a satisfactory explanation based on radiational exchange with the surface of the earth is quite improbable considering that the 5-yr. January mean relative topography for the 50- to 100-mb. layer (Wege [17]) shows warm centers over Kamchatka and eastern Canada and minimum temperatures at the same latitude over England.

In the 1958 case, waves 2 and 3 had the conventional moderate vertical slope toward the northwest, but wave 2 like wave 1, amplified with height while wave 3 diminished with height. Wave 4, with the same amplitude at the two levels, sloped to the north at low latitudes but had a reverse slope to the east at high latitudes.

Waves 5–8 at 500 mb. (fig. 9) are presented largely as a curiosity. Since the members of this group tend to progress eastward (fig. 12), their monthly mean position may be considered as representing the locality of maximum amplitude of the traveling wave. Wave 6 was the only 500-mb. wave with its center north of 55° N. Waves 5 and 6 both retained substantial strength, and the interrelation of their phase position is of some importance. In the half hemisphere centered on the Great Lakes they were mutually reinforcing while in the Eurasian sector they tended to interfere with one another.

4. DAILY PHASE AND AMPLITUDE CHANGES

Spectral analysis of daily charts at both stratospheric and tropospheric levels provides an opportunity to check for interconnections between the levels. The method of presentation used is suggested by that of Kubota and Iida [5], who traced 500-mb. waves in January of both 1946 and 1949 and in the period May to July 1951. The charts presented here also contain a representation of wave amplitude to help in showing the association of large amplitude with regularity of movement and of vanishing amplitude with sudden discontinuity in phase position. When the daily phase movement of a particular wave number is in the neighborhood of a half wavelength, the direction becomes arbitrary; generally in these cases the amplitude is trivial.

The incorporation of phase positions at two different levels in the chart for a given wave number aids in estimating the degree of association between levels. Although for any wave number there are n ridges and n troughs, the phase position for only one ridge is generally presented in each of the diagrams. The day-to-day movement of the ridge selected is shown as a continuous line, but recourse to an adjacent ridge is occasionally made to assist in interlevel comparison.

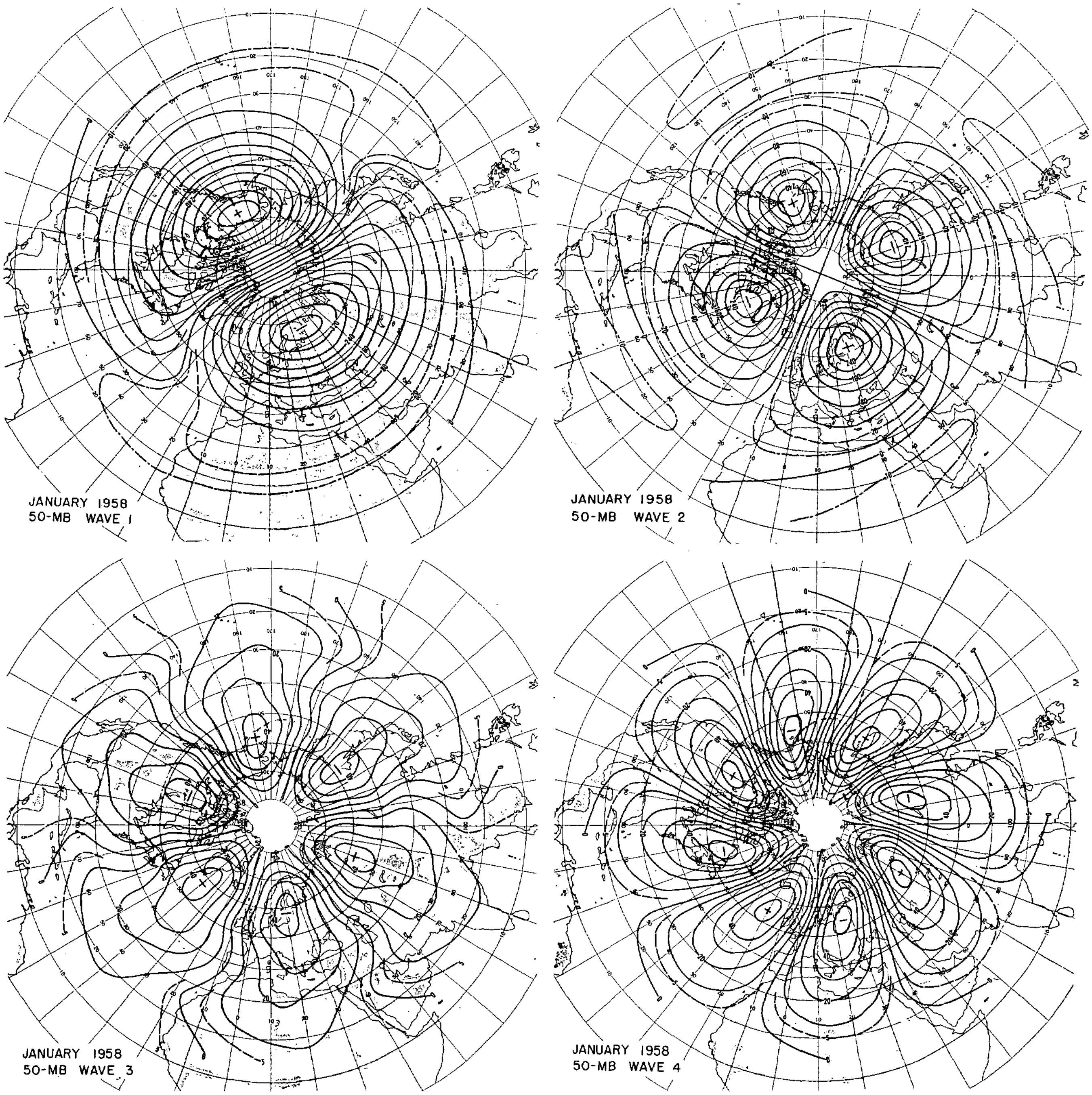


FIGURE 7.—January 1958 50-mb. standing waves 1–4, from Fourier resolution of the monthly mean contour chart, in meters.

Phase and amplitude at 65° N. (fig. 10).—Until January 13 the ridge of wave 1 at 50 mb. moved slowly westward toward Siberia with little change in amplitude. By the 18th it had re-established itself with great amplitude near 150° W. and beginning on the 22d moved eastward for several days.

Between the 16th and 17th the 500-mb. wave 1 ridge apparently crossed the pole to western Canada in response to the sudden amplification at 50 mb. but then drifted eastward to the Iceland area. The period January 9–13 when wave 1 at 50 mb. lay 60° to the east of its 500-mb. position contrasts with the period after the 22d when there was a tendency for a 120° westward slope (found also in the monthly mean position of wave 1 in figs. 7 and 8).

Wave 2 at 50 mb. which was relatively stationary during most of the month amplified rapidly on the 25th. Except during the period January 17–24, this wave sloped upward about 15° toward the west between the two levels. Waves 3 and 4 generally had a similar slope. Like wave 2, they

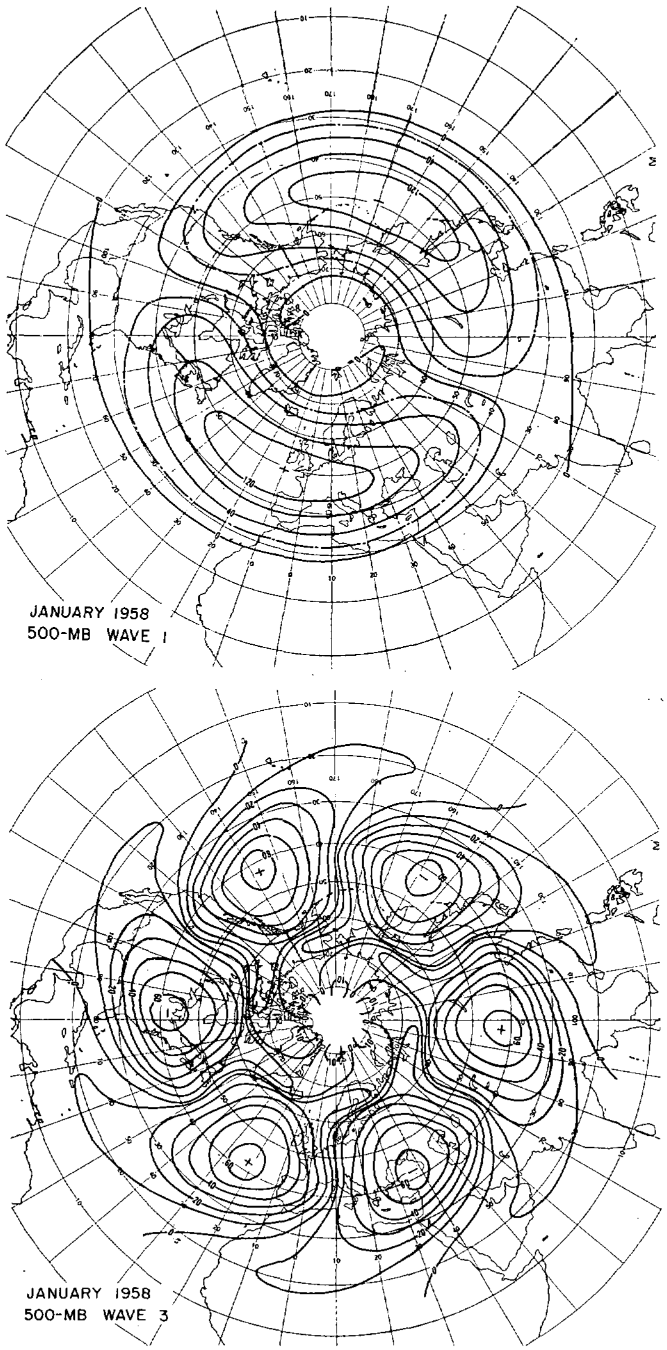


FIGURE 8.—January 1958 500-mb. standing waves 1–4, as in figure 7.

moved alternately eastward and westward with occasional discontinuities in position during periods of low amplitude.

Phase and amplitude at 45° N., waves 1–4 (fig. 11).—Although the 500-mb. wave 1 held fast during the month near the Greenwich meridian, the 50-mb. wave 1 began a long westward excursion in mid-January during the period when wave 1 at 65° N. suddenly intensified.

After amplifying in mid-month, wave 2 tended to maintain a westward slope of about 30° between levels. At 50 mb. waves 1 and 2 both showed a tendency for discontinuous retrogression. Wave 3 had an irregular slope; of some interest is its westward movement in the latter half of the month. In contrast wave 4 had a more regular slight slope and like wave 1 held stationary; furthermore the ridge transpolar from the one diagrammed stayed very close to the ridge of wave 1 at 500 mb., suggesting some sort of relation between the two.

Phase and amplitude at 45° N., waves 5–8 (fig. 12).—At 500 mb. waves 5–8 were all progressive except for some deceleration late in the month particularly by wave 8. Totally different motions occurred at 50 mb. where wave

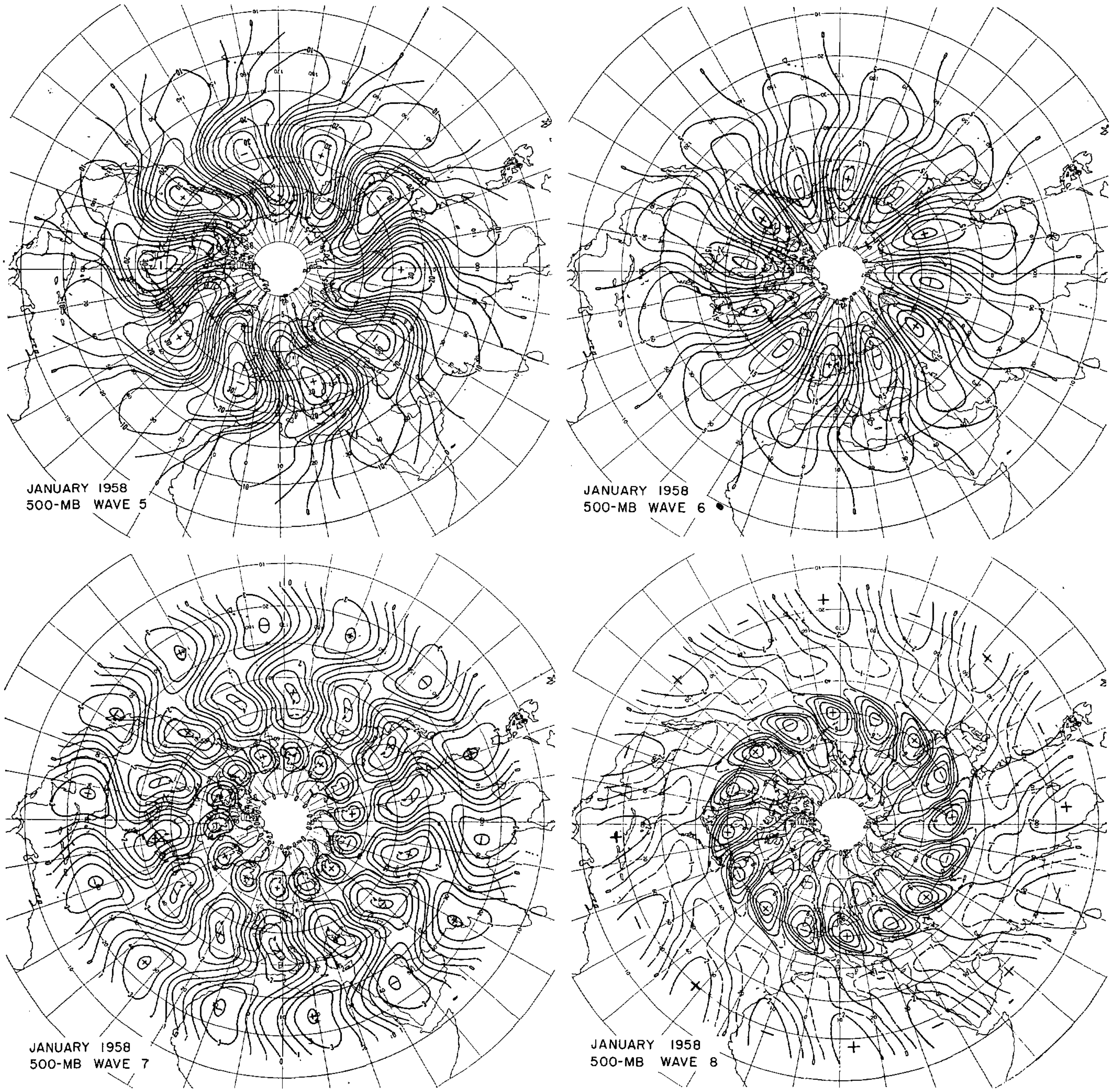


FIGURE 9.—January 1958 500-mb. standing waves 5–8, as in figure 7.

5 retrogressed rapidly, waves 6 and 7 progressed slowly, and wave 8 was practically stationary.

The phase movements shown in figures 10–12 indicate that waves 1–4 formed a family related over altitude and latitude although not always in a simple geometrical way. Within this group, correspondence in movement and constancy of slope increased with wave number. Their movement tended to be quasi-stationary or oscillatory. The 50-mb. waves 5–8 at 45° N. had a similar type of movement and thus seem to have had some connection with waves 1–4. On the other hand the 500-mb. waves 5–8 at 45° N. belonged to a different family moving at nearly the same progressive rate. This rate is approximately that given by the Rossby wave speed equation so that this category of wavelengths in the troposphere might appropriately be designated as Rossby waves.

Several other aspects of the motions of the waves require investigation. For example, how can the retrogression of the 50-mb. wave 5 be reconciled with the fact that no other wave exhibited this behavior? or how can the 50-mb. wave 8 show regular retrogressive movement for a week or more at a time when it does not seem to be

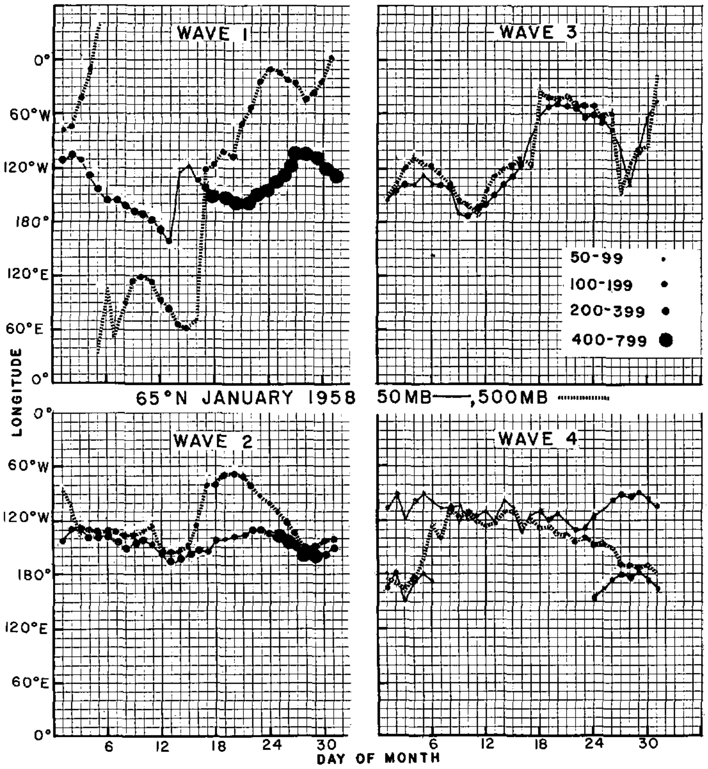


Figure 10.—January 1958 65° N. daily phase and amplitude of waves 1–4 at 50 mb. (solid curve) and 500 mb. (dashed curve). Amplitude code is in meters.

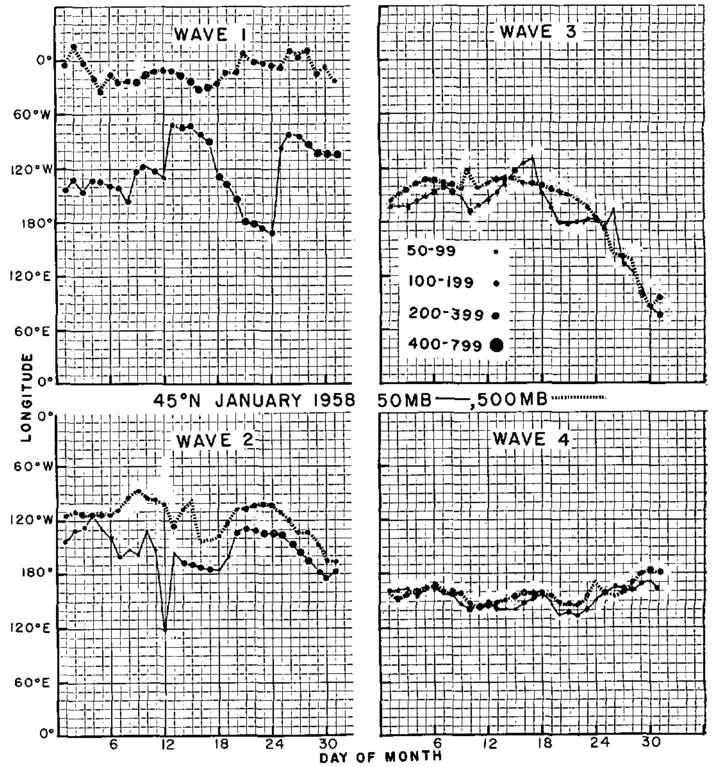


Figure 11.—January 1958 45° N. daily phase and amplitude of waves 1–4 at 50 and 500 mb. Amplitude code in meters.

a real entity on the map? Some of these waves probably existed only as shape parameters of longer waves. Another possibility is that one or more were strongly influenced by diurnal effects, orography, or regional radiosonde error.

5. CORRELATION OF WAVE NUMBER STATISTICS

Because of a finding by Boville [1] that in the 1958–59 winter there was a correlation coefficient of 0.68 between the daily values of kinetic energy in wave 2 at 25 mb. and at 500 mb., a number of wave number quantities have been similarly examined here. As a first step the daily values of wave-2 amplitude at 50, 100, and 500 mb. were plotted at 5° latitude intervals and analyzed to bring out possible relationships (fig. 13). The close association of the 50- and 100-mb. amplitudes is apparent. The observed increase of amplitude with height extended to at least the 30-mb. level where on January 5 the maximum amplitude was 319 m. versus 267 m. at 50 mb.; on January 15, 581 m. versus 396 m.; and on January 25, 578 m. versus 502 m. The attempt to extend the relation down to 500 mb. is frustrating, for although separate centers seem to match from stratosphere to troposphere at the high latitudes, the relationship is vague. However at all levels there was an obvious long term decrease of amplitude from January to February.

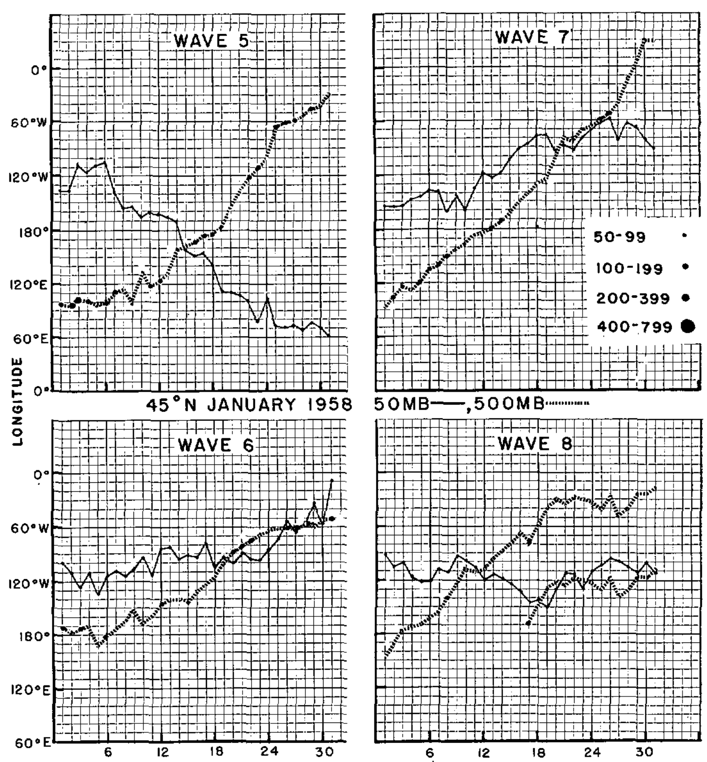


Figure 12.—January 1958 45° N. daily phase and amplitude of waves 5–8 at 50 and 500 mb. Amplitude code in meters.

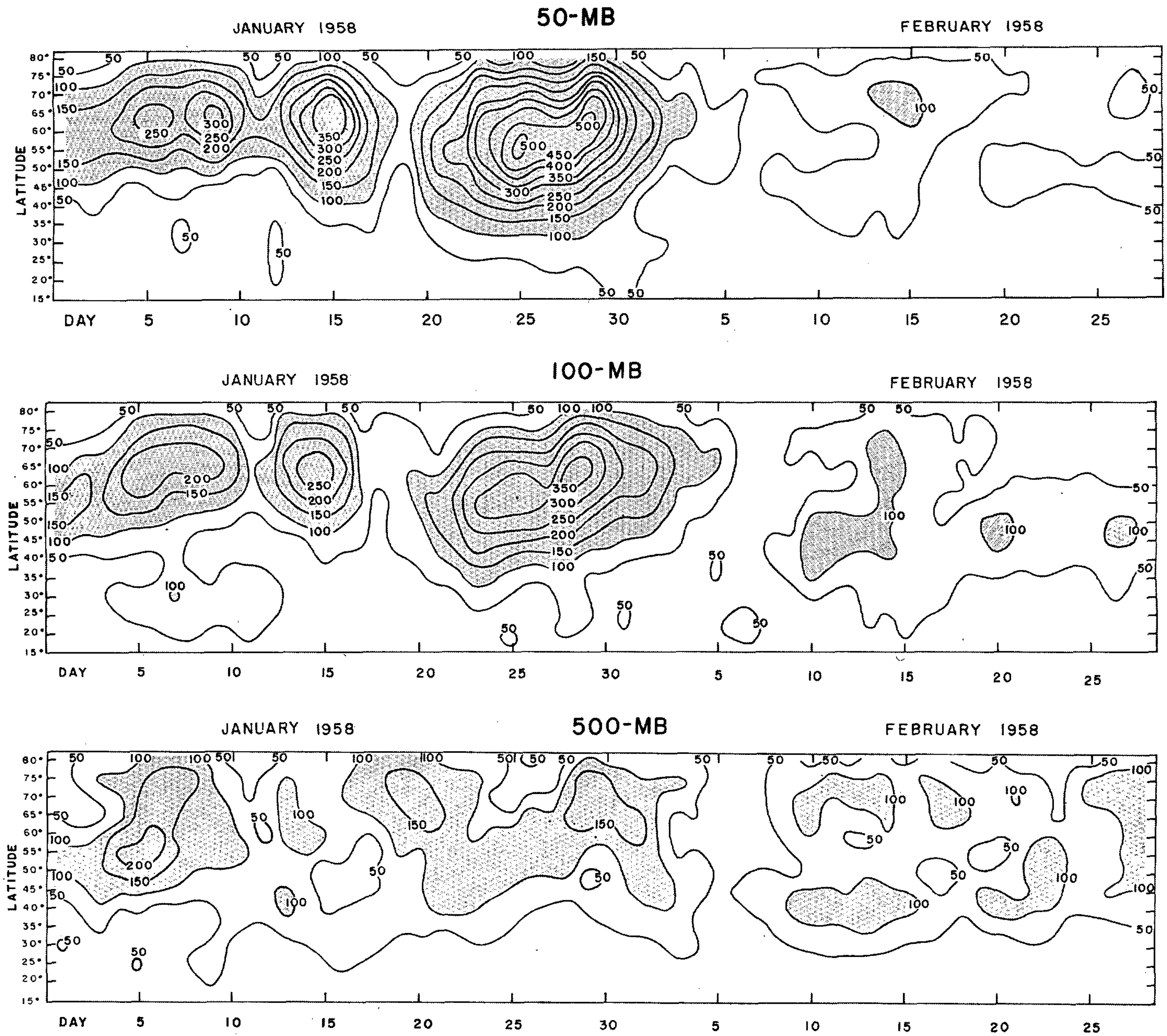


FIGURE 13.—January–February 1958 daily amplitude of wave 2 by latitude at 50, 100, and 500 mb., in meters.

In a further attempt to find interwave, interlatitude, and interlevel relationships, the amplitudes of waves 0–8 at 45° and 65° N. at 50 and 500 mb. for the 90 days of December 1957, January and February 1958 have been processed through IBM 7090 and STRETCH screening programs developed by M. Frankel and H. R. Glahn. One portion of this program was used to produce a matrix of the 630 correlations between all pairs of the 36 variables. If half of the days are taken to be independent ($(N/2)-2=43$ degrees of freedom), with the use of Student's "t", significance levels can be defined for the correlation coefficient: $r=t/\sqrt{43+t^2}$. On this basis the correlation required for the 5 percent significance level is 0.29, for 1 percent significance 0.38, and for 0.1 percent significance 0.47.

The correlations among amplitudes of individual waves at each latitude and level are presented in table 1. Because of a suspicion that the large number of significant correlations by the above rule is a spurious result due to great interdiurnal persistence (i.e., far less than half of the days are independent), the seasonal trend has been removed in large part by finding the 24-hr. changes in amplitude and repeating the correlation procedure with these "pre-whitened" data. After pre-whitening [1] (table 2) there is a drastic reduction in the number of significant correlations. For example, the daily amplitudes of wave

[1] The term "pre-whitening" refers to the removal of low-frequency variation such as the seasonal trend for the purpose of flattening the curve of the time series in order to emphasize the variation in the wave band of interest. This process has also been referred to as the removal of red noise.

Table 1.—*Correlations among amplitudes by wave number at 45° and 65° N. and at 50 and 500 mb. (Dec. 1957–Feb. 1958).*

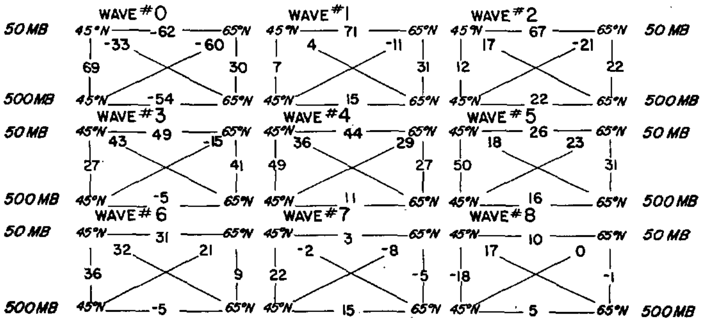


Table 2.—*Correlations among 24-hr. changes in amplitude for each wave number for 45° and 65° N. and at 50 and 500 mb. (Dec. 1957–Feb. 1958).*

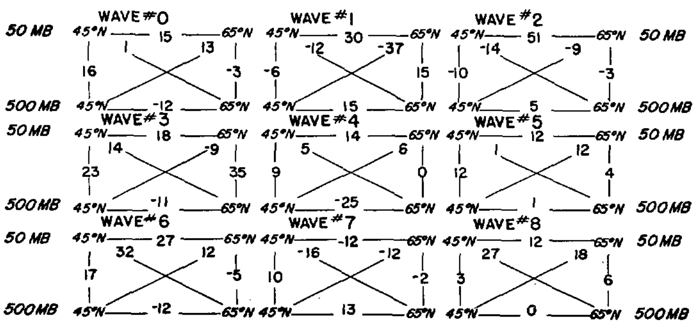


Table 3.—*Interlevel correlations by wave number of total kinetic energy (in 10^{21} ergs mb.$^{-1}$) in the zone 17.5°–77.5° N., December 1957–February 1958. $N=90$*

Wave Number	Interlevel Correlation Pressure Level (mb.)			Mean and Standard Deviation					
				50 mb.		100 mb.		500 mb.	
	50–100	50–500	100–500	Mean	σ	Mean	σ	Mean	σ
n=0	0.28	−0.41	0.45	1860	898	4191	452	1998	212
1	.66	−.06	.46	471	327	404	151	302	113
2	.93	.03	.23	311	334	240	161	194	88
3	.73	.05	.52	138	78	195	95	267	127
4	.62	.57	.77	88	67	138	76	196	115
5	.76	.66	.73	48	33	120	109	168	110
6	.48	.37	.57	25	16	85	58	152	81
7	.36	.20	.56	13	10	46	31	113	70
8	.16	−.08	.29	8	5	27	20	99	50

Table 4.—*Interlevel correlations by wave number of interdiurnal change in total kinetic energy (in 10^{21} ergs mb.$^{-1}$) in the zone 17.5°–77.5° N., December 1957–February 1958. $N=89$*

Wave Number	Interlevel correlation Pressure Layer (mb.)			Mean* and Standard Deviation					
				50 mb.		100 mb.		500 mb.	
	50–100	50–500	100–500	Mean	σ	Mean	σ	Mean	σ
n=0	0.13	0.03	0.26	9.72	145	6.46	280	2.75	99
1	.34	−.13	.36	.72	119	−.53	88	2.01	78
2	.62	.01	.23	−.02	148	−.28	83	1.49	54
3	.57	.04	.41	.31	63	.20	76	1.06	103
4	.63	.32	.40	−.37	54	−1.16	63	−.43	66
5	.48	.35	.24	−.07	26	0	92	−.04	74
6	.29	.22	.24	.29	20	1.94	51	1.13	64
7	.12	−.09	.41	.03	12	−1.16	28	−.65	70
8	.19	.00	.23	−.04	6	−.79	20	0	56

* The mean interdiurnal change is the total kinetic energy on February 28, 1958 minus that on December 1, 1957, divided by 89.

2 at 50 and 500 mb. have a correlation of 0.22 at 65° N. and 0.12 at 45° N. For 24-hr. changes in wave-2 amplitude, the corresponding figures are −0.03 and −0.10. If all of the 89 differences are considered independent for the pre-whitened data ($N-2=87$ degrees of freedom), the correlation for the 5 percent significance level is 0.21, for 1 percent significance 0.27, and for 0.1 percent significance 0.34.

For the pre-whitened data wave 3 exhibits the largest correlations between the 50- and 500-mb. levels at a single latitude, 0.23 at 45° N. and 0.35 at 65° N. The most significant of the remaining correlations is the −0.37 for the 50-mb. wave 1 at 65° N. vs. the 500-mb. wave 1 at 45° N. This negative correlation is to be expected in view of the out-of-phase relation and latitudinal distance between the wave-1 centers at the two levels (compare figs. 7 and 8).

As a further check on possible relations between waves at different levels, a correlation was run on the total kinetic energy in the zone from 17.5° N. to 77.5° N. for each wave number 0–8 at 50, 100, and 500 mb. respectively (tables 3 and 4). Whereas Boville [1], using total kinetic energy in the 40°–80° N. zone during the 1958–59 winter, found high correlation for wave 2 at 25 mb. and at 500 mb., we find significant correlations for waves 4, 5, and 6 at 50 mb. and at 500 mb. Because the 17.5°–77.5° N. zone is almost twice the size of the 40°–80° N. zone (so that its kinetic energy at 500 mb. is more heavily influenced by the polar front jet stream) our correlations of kinetic energy are not strictly comparable with those of Boville. However, even our correlations of amplitudes at two different latitudes with the 40°–80° N. zone (tables 1 and 2) show no special role for wave 2 in the December 1957–February 1958 period.

A casual inspection of daily values of the spectral energy function[2] L suggested the existence of a negative relation between values for adjacent wave numbers; for example, at 500 mb. in January 1958 wave 5 tended to gain energy from the other eddies as a group on the same days that wave 6 was losing energy to the other waves. To study this relationship more completely and also to test for a relationship between kinetic energy received by a given wave from all other waves and that received from the mean flow, $-M$ values[3] as well as L values for the wave numbers 1–8 were intercorrelated in the same way as the amplitude values. The correlations between

[2] $L(n)$ is the spectral function for the transfer of kinetic energy into the eddies of wave number n from all other eddies due to interactions of horizontal winds (Saltzman and Fleisher [11]).

[3] $M(n)$ is the spectral function for the transfer of kinetic energy into the mean zonal motion from eddies of wave number n due to the action of horizontal Reynolds stresses on the zonal current (Saltzman and Fleisher [10]). The correlations are referred to $-M$ in order to retain the same sense of energy transfer as for L; i.e., into eddies of wave number n.

TABLE 5.—*Correlations by wave number among exchange processes represented by spectral energy functions L and M at 50 and 500 mb. (Dec. 1957–Feb. 1958).*

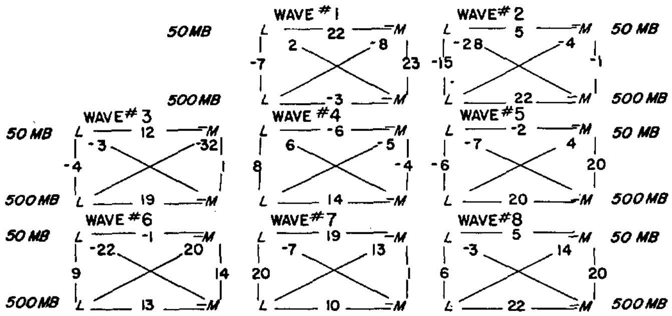
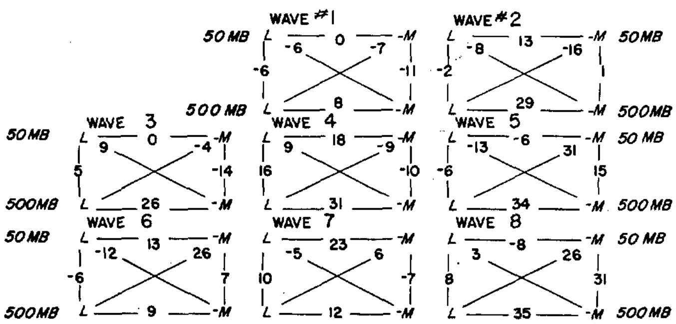


TABLE 6.—*Correlations by wave number among 24-hr. changes in exchange processes at 50 and 500 mb. (Dec. 1957–Feb. 1958).*



L and $-M$ for individual wave number (tables 5 and 6) tend to be positive at 500 mb.; i.e., the day-to-day transfer of kinetic energy from a wave to the zonal flow ($-M$ negative) tends to accompany transfer from that same wave to the other waves (L negative) and vice versa.

A clear tendency for negative correlation between L's for consecutive wave numbers (table 7) holds up very well even when the 24-hr. changes in L are correlated. This suggests a proclivity for the exchange of kinetic energy between adjacent wave numbers; that is, if wave 6 is releasing kinetic energy to other waves, the recipients are most likely to be waves 5 and 7 and vice versa. In general, action in one wave appears to be accompanied by simultaneous reaction in the adjacent wave numbers.

Next the energetics of wave number 1 and of wave number 0, the zonal flow, were compared with respect to their exchange of kinetic energy with waves 2–8. Correlations between the 24-hr. change in kinetic energy entering each of waves 2–8, $L(n)$, with that entering the zonal flow $+M(n)$, and also with that entering wave 1, $L(1)$, are shown in table 8. For each of the 500-mb. wave numbers tested there is a negative correlation between $L(n)$ and $M(n)$; that is, a given wave tends to distribute (receive) kinetic energy simultaneously to (from) both the zonal flow and all other waves as a group. At 50 mb. for this 3-month period similar negative correlations occur only for waves 2, 4, 6, and 7. For the analogous relation

TABLE 7.—*Correlation in energy transfer* (L(n)) *and in 24-hr. change in* L(n) *occurring in adjacent waves, December 1957–January 1958*

Pressure Level (mb)	L(n)							
	Wave Number							
	1	2	3	4	5	6	7	8
50	−.18	−.34	.07	−.20	−.49	−.12	−.19	
500	−.13	.02	−.48	−.11	−.36	−.07	−.18	
	24-HOUR CHANGE IN L(n)							
	1	2	3	4	5	6	7	8
50	−.19	−.20	.18	−.11	−.39	−.02	.23	
500	−.12	.06	−.31	−.18	−.29	−.24	−.11	

TABLE 8.—*Correlation in 24-hr. change in kinetic energy transfer into wave n with that into the zonal flow and into wave 1, December 1957–January 1958*

	24-HOUR CHANGE IN L(n) AT 50 MB.							
	Wave Number							
	1	2	3	4	5	6	7	8
M(n)	0	−13	0	−18	6	−13	−23	8
L(l)	100	−19	−20	−35	−2	10	11	−19
	24-HOUR CHANGE IN L(n) AT 500 MB.							
	Wave Number							
	1	2	3	4	5	6	7	8
M(n)	−8	−29	−26	−31	−34	−9	−12	−35
L(l)	100	−12	−32	−30	−1	−6	2	−7

of waves 2–8 with wave 1, there are sizeable negative correlations between the kinetic energy transfer into wave 1 and that into waves 2, 3, and 4 both at 50 and 500 mb. At 50 mb. where the eccentricity of the flow was very pronounced in this period, the negative correlations between wave 1 and waves 2–4 are somewhat more significant than those between the zonal flow and these long waves. On the other hand, the energy transfer of the shorter waves 5–8 at 50 mb. shows only a weak indication of correlating with that of either the zonal flow ($L(n)$ vs. $M(n)$) or wave 1 ($L(n)$ vs. $L(1)$). At 500 mb. there is an evident negative correlation between the kinetic energy transfer of waves 5–8 with that of the zonal flow but not with that of wave 1.

These results lead to the following conclusions:

1. Consecutive wave numbers tend to exchange kinetic energy with each other (since one tends to be losing kinetic energy when the other is gaining).

2. At least during this period studied, when wave 1 has great average amplitude the longer waves 2–4 at 50 mb. have a greater tendency to exchange energy with wave 1 than with the zonal flow.

3. At 500 mb. all waves 1–8 tend to exchange kinetic

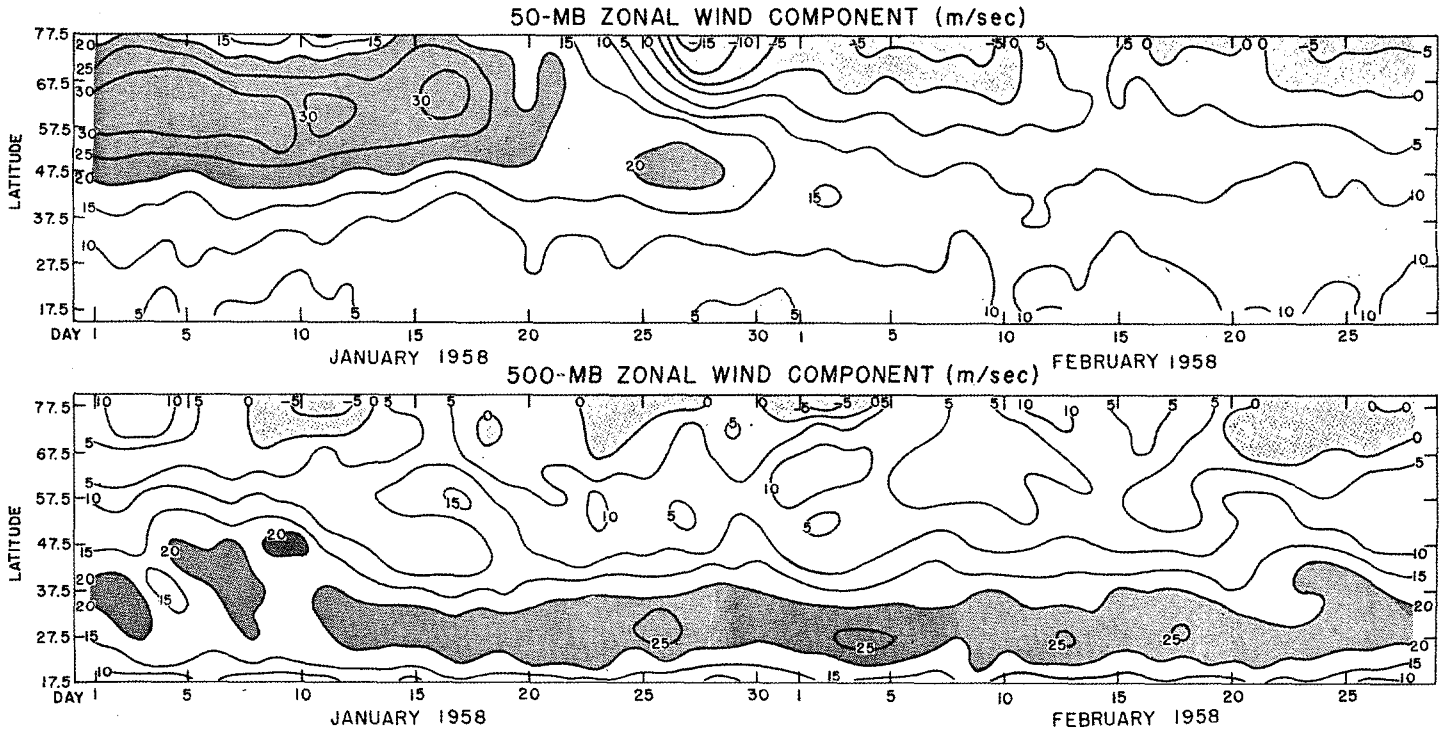


FIGURE 14.—January–February 1958 daily zonal wind component by latitude at 50 and 500 mb. Speeds greater than 20 m. sec^{-1} are shaded.

energy directly with the zonal flow. Waves 2–4 also exchange energy directly with wave 1.

4. Waves 5–8 have a greater tendency to exchange kinetic energy one number up or down scale than directly with wave 1.

6. THE ZONAL FLOW

In a further attempt to find interrelations between the stratosphere and troposphere during the warming epoch, daily values of the zonally-averaged zonal winds $[u]$ at each level have been plotted with respect to latitude. The analyses of these values for the 50- and 500-mb. levels in January and February 1958 (fig. 14) suggest factors deserving further investigation.

On January 10 at 500 mb., the zonal westerlies broke rapidly southward from 45° N. and then became strong and steady at 30° N. At 100 mb. (not shown), a similar break occurred in a westerly belt located near 55° N. and secondary to a stronger subtropical belt of westerlies near 30° N. At 50 mb. the jet flow of about 30 m. sec.$^{-1}$ remained quite steady near 62.5° N. until the beginning of a rapid decay in the high latitudes after January 17. Thereafter, the residual belt of strongest westerlies at 50 mb. tended to remain at lower latitudes.

From the time of its early development in October 1957, the stratospheric jet stream maintained a position about 20° to 25° of latitude north of the belt of strongest 500-mb. westerlies. Thus the southward expansion of the 500-mb. westerlies in the week preceding the breakdown of the stratospheric westerlies suggests that the tropospheric wind belt is able to constrain the stratospheric jet stream to the high latitudes but that the stratospheric wind belt, when allowed to spread southward, begins to meander. If these events at the two levels are connected, then there must have been a chain of associated events in the lag period of a week separating them. A possible answer lies in wave growth alternating between waves 2 and 1 beginning on January 11 (fig. 15). The successively greater amplifications led to a great maximum in total eddy kinetic energy during the last days of the month.

7. KINETIC ENERGY CHANGES

To aid in isolating factors pertinent to the amplification of the stratospheric waves, daily values of some 50-mb. kinetic energy parameters have been assembled in figure 15. The kinetic energy of the zonal flow in the belt from 17.5° to 77.5° N. was at its wintertime maximum of 33.5 units on January 6, having built up from only 2 units (one unit $=10^{23}$ ergs mb.$^{-1}$) on October 4. An irregular moderate decrease thereafter was followed by a rapid drop after January 17. On the same date the total eddy kinetic energy, which had oscillated near 10 units, began a rise to 28.5 units on January 28, when it, too, dropped off rapidly.

A fairly slow and regular increase of the total kinetic energy of the polar stratosphere followed by a rapid dissipation has occurred in other winters. For the previous winter (1956–57) Craig and Lateef [2] report a similar rapid drop in the total kinetic energy of the 25- to 100-mb. layer during the 10 days following February 4, 1957. In

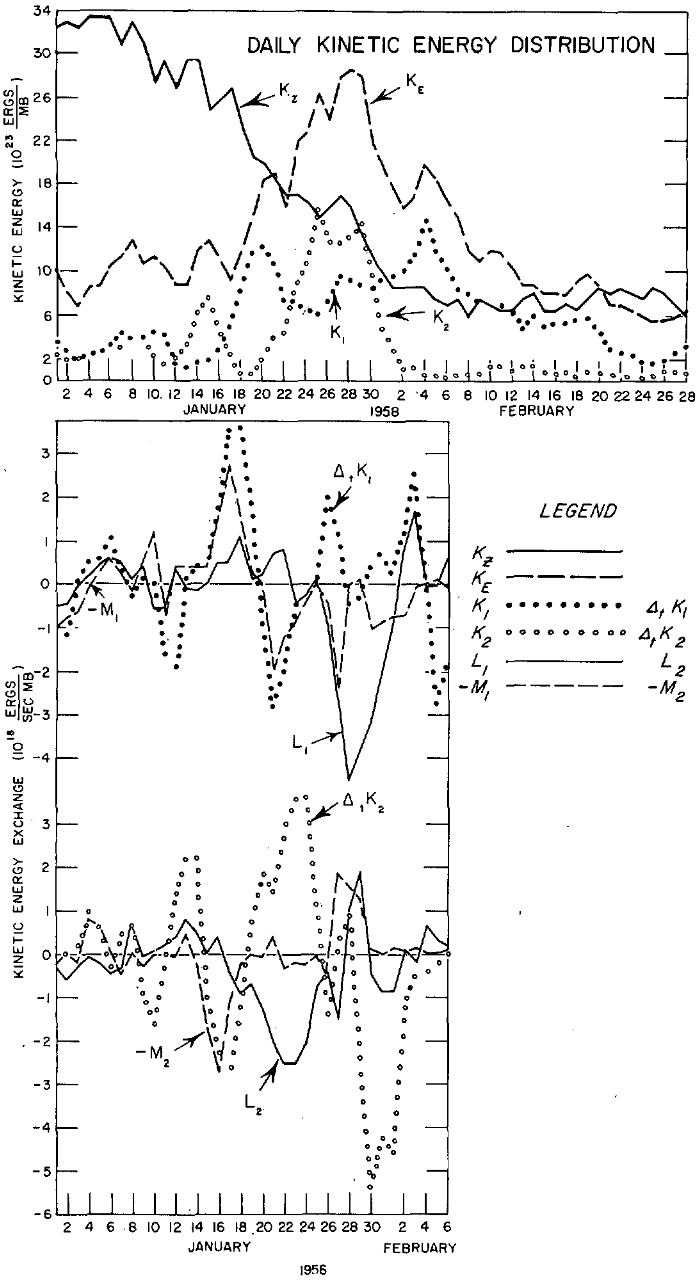


FIGURE 15.—January-February 1958 50-mb. daily kinetic energy distribution in zonal and eddy flow and daily kinetic energy storage and exchange in waves 1 and 2.

this period their calculations show a pronounced conversion of kinetic energy due to vertical transport of geopotential.

For the following year (1958–59) Boville [1] describes a rapid drop in total kinetic energy initiated on February 17, 1959. He reports a small net conversion from eddy kinetic to eddy available potential energy for the month of February 1959 while for January, when the eddy kinetic energy was still accumulating, there was a large energy conversion in the opposite sense.

Since the kinetic energy in waves 3–15 in January and February 1958 varied only a little from day to day in any single wave or in the group as a whole, the plots for just waves 1 and 2 are included in figure 15. The kinetic energy in wave 1 reached a peak of 12.5 units on January 20 and another of 14.5 units on February 4. Meanwhile the kinetic energy in wave 2 rose from 0.5 units on the 19th to 15.5 units on the 25th and dropped equally rapidly from 14.5 units on the 29th to 0.5 on February 4. Of course to some extent this was the same energy being exchanged back and forth between waves 1 and 2 as they alternately waxed and waned.

Separate plots in figure 15 of the kinetic energy changes in waves 1 and 2 respectively, show the eddy variations more clearly. The 24-hr. change in eddy kinetic energy ($\Delta_t K$) and the rates of kinetic energy reception from the other eddies (L) and from the zonal flow ($-M$) are given. There are interesting differences from one amplification to another. For example, wave 1 underwent a major amplification from January 16–19 at a time when $\Delta_t K_1$ was approximately equal to the sum of L_1, and $-M_1$. In this sense the event may be considered as a barotropic amplification, but actually all of the energy conversion and exchange processes taking place at the time have not been computed.

During the period January 19–25, wave 2 underwent a similarly large amplification at 50 mb. but simultaneously dispersed kinetic energy to the other waves (L_2 negative). This is an illustration of probable baroclinic amplification.

Evidently the development of stratospheric systems may involve different combinations of complex interaction among the waves and between the waves and the zonal flow. Although there is a temptation to look upon the breakdown of the westerlies as a single event ascribable to a single causative factor, this study reveals a chain of events each of which appears to set up the energy in an unstable state leading to the next event. In the succession of events or even simultaneously in different scales of motion, more than one type of instability may be involved. Sooner or later the stabilizing gravitational, rotational, and frictional forces gain control, and a new stable state is established.

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